

A Category \mathcal{O} for $\mathfrak{sl}_\infty(\mathbb{C})$

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Abstract

We fix a splitting Borel subalgebra $\mathfrak{b} \subseteq \mathfrak{sl}_\infty(\mathbb{C})$ which is generated by its Cartan subalgebra and the simple root spaces. We then consider the category $\mathcal{O}_\mathfrak{b}^\infty$ defined by \mathfrak{b} . This category differs significantly from the usual category $\mathcal{O}_\mathfrak{b}^\infty$ as the Verma and co-Verma modules have infinite length in general, and some simple objects have no injective hulls.

For this reason, we study truncated subcategories $\mathcal{O}_\mathfrak{b}^\infty$ for which we establish BGG reciprocity. We then show that these local BGG reciprocities lead to a global BGG reciprocity for $\mathcal{O}_\mathfrak{b}^\infty$. In addition, we study the block decomposition of $\mathcal{O}_\mathfrak{b}^\infty$ and Kazhdan-Lusztig multiplicities.