Problem 1. Let $F$ be a field of characteristic $\text{char}(F) = p > 0$, $f : V \rightarrow V$ be an endomorphism of a finite-dimensional $F$-vector space $V$.

1. If $\text{tr}(f^n) = 0$ for $1 \leq n \leq \dim_F V$ and $f$ is not nilpotent, then $p \mid \dim_F V$.
2. Let $L$ be a solvable Lie algebra such that $\dim_F L \leq p - 1$. Then $[L, L]$ is nilpotent.

Problem 2. Let $L$ be a Lie algebra over $F$, $V$ be a finite-dimensional irreducible $L$-module.

1. If $f : V \rightarrow V$ is an endomorphism of $L$-modules, then $f = 0$ or $f$ is invertible. (Schur’s Lemma)
2. If $F$ is algebraically closed, then $\text{End}_L(V) = \{ \alpha \cdot \text{id}_V : \alpha \in F \}$.
3. If $F$ is algebraically closed and $x \in C(L)$, then there exists $\alpha(x) \in F$ such that $x.v = \alpha(x)v \ \forall \ v \in V$.

Problem 3. Let $L$ be a finite-dimensional Lie algebra such that every finite-dimensional $L$-module $V$ affords a weight space decomposition $V = \bigoplus_{\lambda \in \Lambda_M} V_\lambda$ ($\Lambda_M \subseteq \text{Map}(L, F)$) with $L$-invariant weight spaces $V_\lambda$. Show that $L$ is nilpotent.

Problem 4. Suppose that $\text{char}(F) = p > 0$ and let $V$ be an $F$-vector space with basis $\{ v_0, \ldots, v_{p-1} \}$.
Consider the linear maps $x, y : V \rightarrow V$, given by
$$x(v_i) := iv_{i-1} \ ; \ y(v_i) := v_{i+1} \quad 0 \leq i \leq p - 1,$$
where $v_{-1} := 0 =: v_p$.

1. Show that $\mathfrak{h} := Fx \oplus Fy \oplus F \text{id}_V$ is a nilpotent subalgebra of $\text{gl}(V)$.
2. Show that $V$ obtains the structure of an irreducible $\mathfrak{h}$-module via $h.v = h(v)$ for $h \in \mathfrak{h}$ and $v \in V$. 