Unconditional bases for spaces with Jacobi weights

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Abstract

In [2], the author constructed a polynomial Schauder-basis $(p_{\alpha,\beta,n})_{n\in\mathbb{N}_0}$ of optimal degree with Jacobi-orthogonality. In particular, it was shown that for all $\varepsilon > 0$ and all $\alpha, \beta \geq -\frac{1}{2}$ with $\max\{\alpha, \beta\} > -\frac{1}{2}$ there exists a sequence of polynomials $(p_{\alpha,\beta,n})_{n\in\mathbb{N}_0}$, such that

1. $\displaystyle\int_{-1}^{1} p_{\alpha,\beta,j}(x)p_{\alpha,\beta,i}(x)(1-x)^{\alpha}(1+x)^{\beta}\mathrm{d}x = \delta_{ij},$
2. $\deg p_{\alpha,\beta,n} \leq n(1+\varepsilon)$ for all $n \in \mathbb{N}_0$,
3. $\forall f \in C[-1,1] : \|f - S_n f\|_\infty \to 0 \qquad (n \to \infty),$
4. $\|S_n\|_{C\to C} \leq c\left(\frac{1}{\varepsilon}\right)^{2\max\{\alpha,\beta\}+1},$

where $S_n f(x) := \sum_{j=0}^{n} c_j(f)p_{\alpha,\beta,j}(x)$ with $c_j(f) := \int_{-1}^{1} p_{\alpha,\beta,j}(t)f(t)\omega_{\alpha,\beta}(t)\mathrm{d}t$.

Combining refined asymptotic estimates from [2], atomic decomposition methods for weighted Hardy spaces and proof structures from the works of Wojtaszczyk and Woźniakowski [1] (rooting back to the famous H^1-results from Carleson), we are able to show that the same construction leads to an unconditional basis for the spaces $L^p_{\alpha,\beta}$, $1 < p < \infty$, and $H^1_{\alpha,\beta}$.

References

[1] Wojtaszczyk, P. and Woźniakowski, K. Orthonormal polynomial bases in function spaces.. *Isr. J. Math. 75:2-3* (1991), 167–191.

[2] Schnieder, J. Gradoptimale Schauder-Basen mit Jacobi-Polynomen. *Dissertation, Universität zu Lübeck* (2010).