Numerical Integration, Discrepancy and Negative Dependence

Marcin Wnuk

Let $d, N \in \mathbb{N}$. Discrepancy of a finite point set $P = (p_j)_{j=1}^N \subset [0,1)^d$ is a way of measuring, how far is the normalized counting measure supported on P away from the Lebesgue measure on $[0,1)^d$. Loosely speaking it turns out that point sets with low discrepancy yield good nodes for quasi-Monte Carlo quadratures, i.e. quadratures of the form

$$Q(f) = \frac{1}{N} \sum_{p \in P} f(p).$$

Motivated by this fact we are looking for randomized point sets which with high probability achieve low discrepancy. To this end we introduce the concept of negative dependece. A randomized point set P is negatively dependent if, intuitively speaking, points from P do not tend to cluster more as in the case if they were independent.

In my talk I give a short introduction to the discrepancy theory and show how the negative dependence property of a randomized point set may be employed to obtain favorable discrepancy bounds. Furthermore, I provide a few examples of negatively dependent point sets and discuss some propagation properties of negative dependence.

No prior knowledge of discrepancy theory or numerical mathematics is required.