Mathematisches Seminar der CAU
Ludewig-Meyn-Straße 4
24098 Kiel

Mathematisches Kolloquium

Freitag, den 15. November 2019

16 Uhr c.t.

Es spricht

Professor Joel Spencer
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New York University, USA

über das Thema

Combinatorics, Probability, ... and Logic

Abstract:
Logic adds an additional dimension to the study of random combinatorial structures, seeking results for ALL properties expressible in a given logical structure.

In the classic Galton-Watson tree (each node having Poisson mean λ children) the property of being infinite is given by a tree recursion: T is finite iff all subtrees $T(v)$, v a child of the root, are finite. This led Galton and Watson to the classic equation

$$1 - y = e^{-y\lambda}$$

for the probability y of being infinite. They saw that with $\lambda > 1$ this has two solutions: $y = 0$ (we're doomed) and $y = y_0 > 0$ (no we're not). Fortunately (?) $y_0 > 0$ is the correct solution. What about "some node has precisely one grandchild"? One can find (try it!) a system of two equations in two unknowns yielding the probability.

This time, however, the equations have a unique solution. Moreover, the solution is the fixed point of a contraction on a compact space.

Why the difference? Grandchild is expressible using quantification over vertices while Infinite is not. (For logicians: the distinction between First Order and Monadic Second Order.) We show that any property expressible using only quantification over vertices behaves like Grandchild.

More complex logics give examples with more complex behavior. Con- sider "There exists an infinite full binary subtree, beginning at the root." One gets an equation with *three* solutions, which have very distinct properties.

Bonus: The Ehrenfeucht Game plays a key role!

Kaffee und Tee im Konferenzraum (423) um 15.30 Uhr.

Zu dieser Veranstaltung laden ein
DIE PROFESSOREN UND DOZENTEN DER MATHEMATIK