Project Summary

This project seeks to apply Lie theoretic and geometric methods to the study of module categories affording tensor products. The primary focal points are the representation types and the stable Auslander-Reiten quivers of the relevant Frobenius categories.

Given a finite group scheme $\mathcal{G}$ over a field k of characteristic $p > 0$, the Friedlander-Suslin Theorem ensures that the even cohomology ring $\mathrm{H}^\bullet(\mathcal{G}, k)$ is finitely generated. Moreover, for each finite-dimensional $\mathcal{G}$-module M, the algebra homomorphism

$$\Phi_M : \mathrm{H}^\bullet(\mathcal{G}, k) \longrightarrow \mathrm{Ext}^*_{\mathcal{G}}(M, M) \;\; ; \;\; [f] \mapsto [f \otimes \mathrm{id}_M]$$

endows the Yoneda algebra $\mathrm{Ext}^*_{\mathcal{G}}(M, M)$ with the structure of a finite $\mathrm{H}^\bullet(\mathcal{G}, k)$-module. The zero locus of $\ker \Phi_M$ is the *cohomological support variety* of M.

Support varieties and rank varieties provide invariants of the connected components of AR-quivers, and they also are an indispensable tool in the classification of representation-finite and tame blocks of finite group schemes. The recently defined Π-supports of modules lead to refinements, whose ramifications are only emerging. The interaction between combinatorial data given by AR-components and π-points as well as the investigation of new classes of modules with certain sets of Jordan types should shed new light on questions that cannot be tackled by varieties only. For certain groups of tame representation type, a classification of their indecomposables is intended, thus furnishing a solid testing ground for general conjectures involving π-points.

In special contexts, such as representations of Frobenius kernels of reductive groups, additional tools are available. Here reduction techniques based on the highest weight categories of G_rT-modules will be employed in the study of certain classes of modules, such as those affording good filtrations.

Earlier work indicates the importance of the representation theory of infinitesimal groups of height one. In this context, schemes of tori will be studied with the hope of obtaining a better understanding of the structure of Lie algebras possessing simple modules with small varieties.

Topics that are related directly or via the methods involved pertain to periodic modules and singularities of quotients of nullcones, toral stabilizers and the Chevalley restriction theorem, representations of reduced enveloping algebras of restricted Lie algebras, as well as free Lie algebras and Solomon's Descent Algebra.