A CHARACTERIZATION OF UNIFORM CONVERGENCE FOR STOCHASTIC SEMIGROUPS

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ABSTRACT. We consider strongly continuous semigroups $(T_t)_{t\in[0,\infty)}$ of stochastic operators on L^1-spaces (over σ-finite measure spaces) and discuss a characterization of uniform convergence of T_t as $t \to \infty$. If the semigroup $(T_t)_{t\in[0,\infty)}$ is irreducible and A denotes its generator, the following assertions are equivalent:

(i) $\lim_{t\to\infty} T_t$ exists with respect to the operator norm.
(ii) For some $t_0 \in [0,\infty)$ the operator T_{t_0} dominates a non-trivial integral operator, and 0 is a pole of the resolvent of A.
(iii) For some $t_0 \in [0,\infty)$ the operator T_{t_0} dominates a non-trivial integral operator, and the adjoint semigroup $(T_t')_{t\in[0,\infty)}$ on L^∞ is irreducible.

This is interesting for applications to Markov processes and transport equations, and it is also interesting from a theoretical point of view since none of the implications "(i) ⇒ (iii) ⇒ (ii) ⇒ (i)" is trivial.

In our talk, we recall the relevant terminology used above, we discuss some aspects of the proof and we mention potential applications.

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