Computing the scheme of characters in $\mathrm{SL}(2,\mathbb{C})$

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This is joint work with Joan Porti. This talk is about the computation of the character scheme of a finitely presented group.

Let

$$\Gamma \cong \langle \gamma_1,\ldots,\gamma_n \mid r_1,\ldots,r_l\rangle$$

be a finitely presented group, and let $\pi\colon \mathbb{F}_n \twoheadrightarrow \Gamma$ be the the natural surjection from the free group of rank n onto Γ.

The scheme of representations and characters in $\mathrm{SL}(2,\mathbb{C})$ are denoted respectively by $R(\Gamma,\mathrm{SL}(2,\mathbb{C}))$ and $X(\Gamma,\mathrm{SL}(2,\mathbb{C}))$. The corresponding coordinate rings of functions are respectively the universal algebra of $\mathrm{SL}(2,\mathbb{C})$-representations

$$A(\Gamma) = \mathbb{C}[R(\Gamma,\mathrm{SL}(2,\mathbb{C}))]$$

and the universal algebra of $\mathrm{SL}(2,\mathbb{C})$-characters

$$B(\Gamma) = \mathbb{C}[X(\Gamma,\mathrm{SL}(2,\mathbb{C}))] \cong A^{\mathrm{SL}(2,\mathbb{C})}.$$

The reduction $B(\Gamma)^{\mathrm{red}}$ of $B(\Gamma)$ is precisely the algebra of functions of $X(\Gamma,\mathrm{SL}(2,\mathbb{C}))$ as a variety, but a priory $B(\Gamma)$ does not need to be reduced.

For $\gamma\in\mathbb{F}_n$, denote by $t_\gamma \in B(\mathbb{F}_n)$ the evaluation function at this element. Montesinos-Amibilia and González-Acuña have shown that

$$B(\Gamma)^{\mathrm{red}} \cong B(\mathbb{F}^n)/I^{\mathrm{rad}}$$

where

$$I = (\langle t_{r_s} - 2,\ t_{\gamma_i r_s} - t_{\gamma_i} \mid 1\le s\le l,\ 1\le i\le n\rangle)\ .$$

Explicit computations show that in general $B(\Gamma)\not\cong B(\mathbb{F}^n)/I$.

The aim of this talk is to compute the algebra $B(\Gamma)$:

Theorem 1 *Let $\Gamma \cong \langle \gamma_1,\ldots,\gamma_n \mid r_1,\ldots,r_l\rangle$ be a finitely presented group. Then $B(\Gamma)\cong B(\mathbb{F}^n)/I_\Gamma$ where*

$$I_\Gamma = \langle t_{r_s} - 2,\ t_{\gamma_i r_s} - t_{\gamma_i},\ t_{\gamma_j\gamma_k r_s} - t_{\gamma_j\gamma_k} \mid 1\le s\le l,\ 1\le i\le n,\ 1\le j<k\le n\rangle.$$

All notions will be introduced, and various examples and applications will be given.