

Ergodic theory, multiplicative functions and Möbius disjointness

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Abstract:

In 2010, P. Sarnak formulated the following conjecture: For each zero entropy homeomorphism T of a compact metric space X , we have

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n \leq N} f(T^n x) \mu(n) = 0$$

for all $f \in C(X)$ and $x \in X$. Here μ stands for the arithmetic Möbius function. The conjecture relates (multiplicative) number theory and dynamics, in fact, ergodic theory. During the talk, I will explain why this conjecture is difficult and deep. In particular, I will show some relations between the above Möbius disjointness conjecture and the famous Chowla conjecture from 1965 on the autocorrelations of the Möbius function, including the logarithmic form of both (Tao's results, Frantzikinakis and Host's results). We will also discuss how to use some ergodic theory and operator theory to pass from logarithmic averages to ordinary averages. Finally, I will describe two main strategies used so far to deal with Sarnak's conjecture.