

REMARKS ON LOCALIZED SHARP FUNCTIONS ON CERTAIN SETS IN \mathbb{R}^n

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On \mathbb{R}^n let $f_{\Delta}^{\#}(x)$ and $M^{\Delta}f(x)$ denote the classical dyadic sharp function and dyadic maximal function respectively, that is,

$$f_{\Delta}^{\#}(x) = \sup_{x \in Q \in \Delta} \frac{1}{|Q|} \int_Q |f(y) - f_Q| dy, \quad M^{\Delta}f(x) = \sup_{x \in Q \in \Delta} \frac{1}{|Q|} \int_Q |f(y)| dy,$$

where Δ denotes the collection of all dyadic cubes in \mathbb{R}^n and f_Q is the average of f over Q . Suppose that $f \in L^{p_0}(\mathbb{R}^n)$ for some p_0 . The well-known Fefferman-Stein inequality asserts that if $1 < p < \infty$, $1 \leq p_0 \leq p$, and $f_{\Delta}^{\#} \in L^p(\mathbb{R}^n)$, then $M^{\Delta}f \in L^p(\mathbb{R}^n)$ and

$$(1) \quad \|M^{\Delta}f\|_{L^p(\mathbb{R}^n)} \leq C_n(p) \|f_{\Delta}^{\#}\|_{L^p(\mathbb{R}^n)}$$

Let Ω be a domain in \mathbb{R}^n . Our goal is to define for $f \in L^1_{\text{loc}}(\Omega)$ a localized version $f_{\text{loc}}^{\#}$ of the sharp function which will satisfy the analogue of the Fefferman-Stein inequality. By localized we mean that the cubes which are taken in the definition of $f_{\text{loc}}^{\#}(x)$ are contained in a bounded set $\mathcal{B}_x \subset \Omega$. So one possible definition can be taken as follows. Let $\tau : \Omega \rightarrow (0, \infty)$. For $f \in L^1_{\text{loc}}(\Omega)$ we set

$$f_{\text{loc}, \tau}^{\#}(x) = \sup_{x \in Q \subset \Omega, \ell(Q) < \tau(x)} \frac{1}{|Q|} \int_Q |f(y) - f_Q| dy,$$

where Q is any cube (not necessarily dyadic) and $\ell(Q)$ denotes its side-length. Our aim is to show that for certain sets Ω in \mathbb{R}^n if $\tau(x)$ behaves like $\frac{1}{2^n} \text{dist}(x, \partial\Omega)$, then $f_{\text{loc}, \tau}^{\#}$ satisfies (1). These were obtained by proving modifications of the good lambda inequality.

This talk is based on joint work with Jacek Dziubański.