Let $\Gamma$ be a graph with one internal node $0$ and $n \geq 1$ edges meeting each other at $0$. For $x > 0$ and $i \in \{1, 2, \ldots, n\}$ we let $x = (x, i)$ denote the point on $\Gamma$ located on the $i$'th edge at the distance $x$ from $0$.

Let $X = (X_t)_{t \geq 0}$ be a one-dimensional diffusion on $\mathbb{R}_+$ hitting $0$ with probability one and reflected at $0$. A (homogeneous) diffusion spider $X = (X_t)_{t \geq 0}$ is a continuous strong Markov process on $\Gamma$ which on each edge before hitting $0$ behaves as $X$ before hitting $0$. When reaching $0$ the spider chooses, roughly speaking, an edge with some given probability to continue the movement. A rigorous construction of $X$ can be done using the excursion theory.

In this talk we give an explicit expression for the resolvent kernel of $X$. Using this we study the excessive functions of $X$ and derive the so called glueing condition to be satisfied at $0$. We apply the representation theory (Martin boundary theory) of excessive functions and calculate the representing measure of a given excessive function.

This machinery can be used to solve optimal stopping problems for $X$. As an example we consider Walsh’s Brownian spider where the diffusion $X$ is a reflecting Brownian motion. We focus on some cases where the reward is continuous at $0$ and linear on the edges.

The talk is based on a joint work with Jukka Lempa and Ernesto Mordecki.