

# The zero-two laws for cosine families

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For  $a, b > 0$  consider

$$d(a, b) = \sup_{t \geq 0} |\cos(at) - \cos(bt)|.$$

This talk deals with questions of the following type: *Does the implication*

$$d(a, b) < c \quad \implies \quad d(a, b) = 0$$

*hold for all  $a, b$  and some fixed  $c > 0$ ? And if so, what is the optimal  $c$ ?*

The setting, however, will be far more general as we will allow for *cosine families*  $C(t)$  instead of scalar cosines  $\cos(at)$ . Cosine families can be seen as the counterpart of operator-semigroups for second-order abstract Cauchy problems. The corresponding implication then becomes

$$\sup_{t \geq 0} \|C(t) - \cos(at)I\| < c \quad \implies \quad C(t) = \cos(at)I \quad \forall t \geq 0,$$

where  $I$  denotes the identity operator.

We will discuss recent developments around this question involving techniques ranging from operator theory to Gelfand theory.